The intuitive rule theory: Prospective teachers' tendency to claim *same change in perimeter – same change in area*

Demetra Pitta-Pantazi[1], Pessia Tsamir[2] and Constantinos Christou[3]
[1,3] University of Cyprus, [2] Tel-Aviv University
[1] dpitta@ucy.ac.cy, [2] pessia@post.tau.ac.il, [3] edchrist@ucy.ac.cy

Abstract

The aim of the study is to explore the impact of the intuitive rule Same A – Same B on Cypriot elementary school prospective teachers. The prospective teachers were presented with squares and rhombuses which underwent Additive, Percentage and Multiplicative transformations which changed them into rectangles and parallelograms respectively. The participants were asked to describe how these transformations affected the perimeter and area of the geometrical shapes. The results showed that a number of prospective teachers tended to answer that since the two opposite sides decreased and the other two sides increased, by additive, percentage or multiplicative transformations, then both perimeter and area remained the same. Such arguments are in line with the intuitive rule Same A - Same B.

Key Words

Intuitive Rules Theory, Same A - Same B, Perimeter, Area, Square, Rectangle, Rhombus, Parallelogram.

Introduction

The teaching of mathematics is a complex endeavor with teachers playing a central role. In order to enhance mathematics teaching it is important for teachers to have strong pedagogical and subject matter content knowledge (PCK and SMK a la Shulman, 1986; see also, NCTM, 2000; Tirosh & Graeber, 2003). Teachers also need to understand their students' mathematical reasoning, their frequent errors and not less importantly, *why* they make these errors. Teacher education is an important, first step in the professional development of teachers and one that should offer them the opportunity to familiarize themselves with students' reasoning, typical correct and incorrect solutions. Prospective teachers should also be able to use different theoretical models that may offer an analysis of students' incorrect solutions and even allow them to predict students' errors. A number of theories have been presented within the field of mathematics education in order to explain why students err, for example the theory of intuition in mathematics and science (e.g., Fiscbein, 1987), the theory of conceptual change (e.g., Vosniadou, 2002), the model for the improper use of analogy (e.g., Brousseau, 1997), the theorem-in-action (e.g., Vergnaud, 1998), the intuitive rules theory (e.g., Stavy & Tirosh, 2000). In this paper we chose to analyze Cypriot elementary school prospective teachers' incorrect solutions in the light of the intuitive rules theory and in particular to examine the applicability of the intuitive rule *Same A – Same B*. More specifically, the aim of this study is to investigate the impact of the intuitive rule *Same A – Same B* on Cypriot elementary school prospective teachers' comparisons of perimeters and areas when a square is transformed into a rectangle and a rhombus into a parallelogram.

Theoretical Background

Stavy and Tirosh (2000) have introduced the *theory of the intuitive rules* for analyzing and predicting individuals' erroneous solutions to mathematics and science problems. They argue that very often a number of erroneous solutions given by students to various topics are not content specific but rather driven by external features of the problems, which are silent and not relevant to the problem. They have identified three intuitive rules: *Same A – Same B* (e.g. Tirosh & Stavy, 2000), *More A –More B* (e.g. Zazkis, 1999) and *Everything Can Be Divided* (e.g. Stavy & Tirosh, 1993). In this paper we focus on the intuitive rule *Same A – Same B.*

The intuitive rule *Same A – Same B* was identified in comparison problems in which there are two objects or systems, where one quality or quantity A, fulfils the condition $A_1=A_2$, but differ in another quantity B where $B_1 \neq B_2$. However, when students are asked to compare B_1 and B_2 for which the two given objects or systems fulfill either $B_1<B_2$ or $B_1>B_2$, a common incorrect response to such problems, regardless of the content domain, takes the form *Same A-Same B, i.e.,* "$B_1=B_2$ because $A_1=A_2$." (Tsamir, Tirosh, Stavy, & Ronen, 2000, p.333). An example of this would be that when students are presented with the problem: "Danny saved 15% of the money he earned in January and Betty also saved 15% of the money she earned in January", students tend to claim that "Danny and Betty saved the same amount of money", even though there is no given regarding their salaries, and thus, it is not necessarily true.

According to Stavy and Tirosh (2000) this intuitive rule carries Fischbein's (e.g. 1987) characteristics of "intuitive knowledge". This means that *Same A-Same B* based solutions seem self-evident (individuals perceive statements they made on the basis of this rule as being true and do not need any further justification). Such solutions are given with great confidence and perseverance (they often persist in spite of formal learning that contradicts them). Individuals apply the intuitive rule to diverse situations and very often alternatives are excluded as unacceptable. In this paper we use the intuitive rule *Same A – Same B* to analyze Cypriot elementary school prospective teachers' responses to comparison problems about perimeters and areas of squares and rectangles.

Numerous pieces of research report on students difficulties in solving perimeter and area problems. One of the most frequent misconceptions reported in this literature is the tendency of students to argue that "shapes with the same perimeter have the same area" and vice versa (e.g., Dembo, Levin, & Siegler, 1997; Hirstein, 1981; Hoffer & Hoffer, 1992; Menon, 1998; Reinke, 1997; Tsamir & Mandel, 2000; Walter, 1970; Woodward & Byrd, 1983; Medici, Marchetti, Vighi, & Zaccomer, 2005). Some of these researchers suggested that these responses arise from students misunderstanding of the relationship between the concept of perimeter and area. Stavy and Tirosh (2000), argue that such responses are in line with the intuitive rule *Same A – Same B.* In this paper we report on a study that investigated Cypriot elementary school prospective teachers' responses to comparison perimeter, area problems, when presented with a square that is transformed into a rectangles and a rhombus that is transformed into a parallelogram by additive, percentage and multiplicative changes. More specifically the research questions were:

- Do elementary school prospective teachers tend to provide solutions in line with the intuitive rule *Same A - Same B* when solving *Additive-Comparison-Problems* regarding perimeters and areas?

- Do elementary school prospective teachers tend to provide solutions in line with the intuitive rule *Same A - Same B* when solving *Percentage-Comparison-Problems* regarding perimeters and areas?
- Do elementary school prospective teachers tend to provide solutions in line with the intuitive rule *Same A - Same B* when solving *Multiplicative-Comparison-Problems* regarding perimeters and areas?

The study

The research was conducted with a hundred and thirty three elementary school prospective teachers at the University of Cyprus. Two of the authors of this paper were teaching a Mathematics Education course in this teacher education program. The prospective teachers' level of mathematical understanding varied since not all of them had mathematics as a major subject in higher secondary school or had to take a mathematics matriculation exam in order to get a place in the BA Primary Education Program at the University of Cyprus. However, all of them had mathematics at least as a common core subject in higher secondary school and thus had been taught the mathematics involved in the problems of this study.

The research was conducted with the use of questionnaires. The questionnaires were administered by the two authors of the paper during a 50 minutes session of the mathematics education course that these students were taking. The questionnaires were given in Greek and they included the problems that are reported in this paper.

Here we address prospective teachers' solutions to the comparison problem that required from students to compare the perimeter and area of a square and a rhombus when the shapes were transformed. More specifically the participants were presented with twelve Problems. In Part A of the questionnaire (which included six Problems), prospective teachers were presented with a square which was transformed into a rectangle in the following three ways:

(a) by lengthening the two opposite sides of a square by 6 cm and by shortening the other two sides by 6 cm – Additive Problem

(b) by increasing the length of the two opposite sides of a square by 50%, and reducing the other two sides by the same factor – Percentage Problem

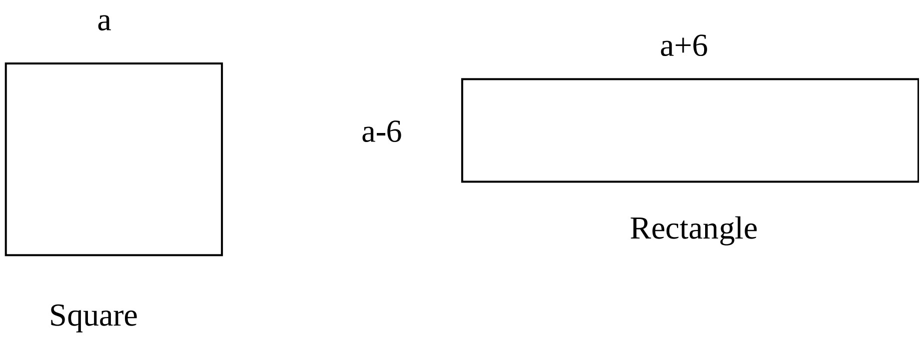
(c) by multiplying the length of the two opposite sides of a square by 6, and reducing the other two sides by the same factor – Multiplicative Problem

In each case the students were asked to compare the perimeter and area of the square with the newly formed rectangle. Figure 1 shows the way in which the Additive (a), Percentage (b) and Multiplicative (c) Problems for Square/Rectangle were presented.

Part B of the questionnaire was comprised by exactly the same questions as Part A, with the difference being that instead of a square the students were presented with a rhombus and they were asked to compare the perimeter and area when this rhombus underwent the three transformations described above (Additive, Percentage, Multiplicative) and became a parallelogram.

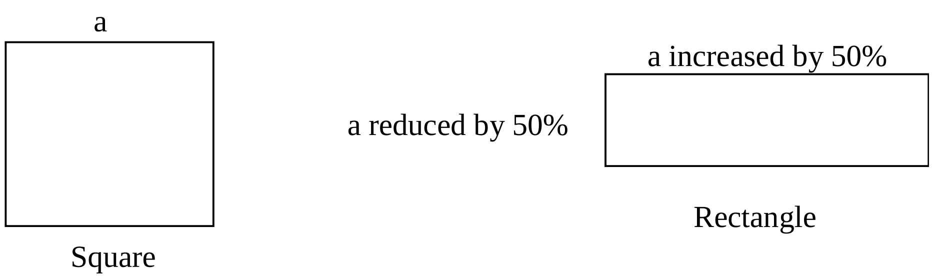
Square/Rectangle Additive Problem

Consider a square, whose sides are “a” cm (a>6cm). A rectangle is created by lengthening the two opposite sides of the square by 6cm, and by shortening the other two sides by 6cm, as described in the drawing.



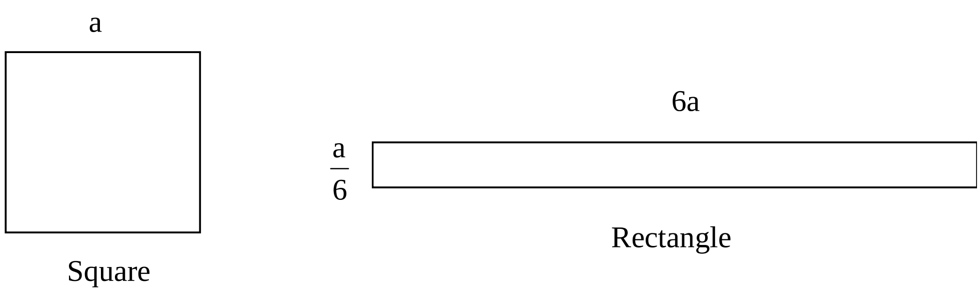
Square/Rectangle Percentage Problem

Consider the same square. A rectangle is created by increasing the length of the two opposite sides of the square by 50%, and reducing the other two sides by the same factor, as described in the drawing.



Square/Rectangle Multiplicative Problem

Consider the same square. A rectangle is created by multiplying the length of the two opposite sides of the square by 6, and reducing the other two sides by the same factor, as described in the drawing.



The perimeter of the rectangle *is larger than/ equal to/smaller than/ impossible to determine* the perimeter of the square. (Circle your choice)

Why? Explain your answer.

The area of the rectangle *is larger than/ equal to/smaller than/ impossible to determine* the area of the square. (Circle your choice)

Why? Explain your answer.

Figure 1: Square/Rectangle Problems

It should be noted that in the Perimeter Additive and Perimeter Percentage Problems, the correct solution is that when the square is transformed into a rectangle and the rhombus into a parallelogram, the perimeter remains the same. Thus, the correct solution is in line with the intuitive rule *Same A – Same B*. However, in the Perimeter Multiplicative Problems the correct solution is not consistent with the intuitive rule *Same A – Same B*. On the other hand, in the Area Problems we have the reverse situation. In the Area Multiplicative Problems the correct solution is in line with the intuitive rule *Same A- Same B,* whereas in the Area Additive and Area Percentage Problems the correct solution does not comply with the intuitive rule *Same A – Same B*.

Results

In this section we present students' responses to the Square/Rectangle and Rhombus/Parallelogram Problems. Both correct and incorrect responses will be presented, with special emphasis to the responses that are in line with the intuitive rule *Same A – Same B*. Tables with the percentage of prospective teachers' correct responses to the problems will be presented as well as indicative examples of the justifications that participants provided for these responses.

The Square/Rectangle Problem

The correct solution to the Perimeter Additive Problem (a) and Perimeter Percentage Problem (b) is that the perimeter of the square and rectangle remain the same. Prospective teachers most often reached the correct answer to both these problems by performing algebraic calculations. However, there appeared to be a greater difficulty in the Perimeter Percentage Problem (88%) than the Perimeter Additive Problem (94%) (Table 1). The reason due to which almost 10% of the participants gave an incorrect response to the Perimeter Percentage Problem was erroneous calculations. For example some participants argued that the *"Perimeter of square =4a, Perimeter of rectangle=(2a+a/2)2=5a", "Perimeter of square=4a Perimeter of rectangle =2(a/2+*a^2*/2)=2(*a^2*)"*.

In regard to the Perimeter Multiplicative Problem (c) the correct answer is that the perimeter of the rectangle is larger than that of the square whereas the areas of the square and of the newly formed rectangle are the same. Participants mainly reached the correct answer by calculations (92%). Several participants (6%) gave the incorrect response that the area and the perimeter of the square and rectangle are equal. Some of them gave this response by arguing that *"I multiply one (side) by 6 and divide the other by 6 therefore the perimeter is the same", "As much as the length of the two sides will increase, that much the length of the other two will decrease"*. Another participant gave a more vague justification that it is *"the same as before"*. These incorrect justifications are based on the intuitive rule *Same A – Same B.*

Table 1, illustrates that the percentage of correct responses decreased when prospective teachers were asked to respond to the Area Problems (94%, 88%, 92% vs 86%, 68%, 91%), with the Area Percentage Problem appearing to be the most difficult one (68%). For the Area Additive Problem, a number of participants (5,3%) gave the incorrect solution that the areas of the square and of the newly formed rectangle were equal. Some of the justifications that the participants provided were *"Because as much as the length of one of the sides is diminished, in the same extent the length of the other side is increased. So when multiplying the sides the result will be the same", "Because the surface of the rectangle is the same with that of the square", "Because* *one side was decreased by 6 and the other was increased by 6, therefore the area remains stable"*. These justifications pointed to *Same A – Same B* based solutions.

In the case of the Area Percentage Problem a number of prospective teachers (13,5%) claimed that the area of the rectangle is larger than the area of the square. Most of them gave this response based on incorrect calculations. Some indicative examples are: *"Area of* square=a^2, *Area of rectangle*=$a/2*3a/2=3/2a^2$, *Therefore Area of rectangle > Area of square"*, *"Area of* square=a^2 Area of rectangle=$a^2/2*a/2=a^3/4$". Some of the participants did not provide any justifications for their answers. About half of the participants who gave incorrect responses (16%) claimed that the area of the square and the area of the rectangle are equal. Several participants' arguments were based on incorrect calculations such as *"Area of square*=a^2, *Area of rectangle*=$2a*a/2=a^2$*"*. Some of the participants' arguments to this problem were consistent with the intuitive rule *Same A – Same B*, for example *"The more the length of the opposite sides increases the more the length of the other two decreases", "Because we increase the two sides and at the same time decrease the other two by using the same number"*.

Although 91% of the participants provided a correct response to the Area Multiplicative Problem, there were several prospective teachers that claimed that the area of the square was larger (4,5%), and some that claimed the opposite (3,8%). Those that argued that the area of the rectangle was larger (4,5%), based their response mainly on incorrect calculations. There was, however, one participant who claimed that the area of the rectangle is bigger since "*One of the sides of the rectangle is bigger than the side of the square"*. This answer is in line with the intuitive rule *More A – More B*. Those (3,8%) who claimed that the area of the rectangle is smaller than that of the square, based their responses on erroneous calculations, which were carried out either with algebraic symbols or more rarely with specific numbers.

SQUARE – RECTANGLE PROBLEMS						
	Perimeter (%) N=133			Area (%) N=133		
Solution	Additive	Percentage	Multiplicative	Additive	Percentage	Multiplicative
Smaller	2,3	0,8	1,5	85,7*	68,4*	3,8
Equal	94*	88*	6	5,3	15,8	91*
Larger	1,5	9	91,7*	5,3	13,5	4,5
Not Determ.	0	0,8	0	3	0,8	0,8
No answer	2,3	1,5	0,8	0,8	1,5	0
** marks the percentage of the correct solution*						

Table 1: Students' responses to the Square-Rectangle Problem

The Rhombus-Parallelogram Problem

Most of the prospective teachers provided correct responses for all the Rhombus Perimeter Problems (Table 2). Almost all of the participants (94%) gave a correct response to the Perimeter Additive Problem, whereas the percentage of correct responses dropped by 10% for both the Perimeter Percentage Problem (82%) and for the Perimeter Multiplicative Problem (83%). The correct responses to the Rhombus Perimeter Problems were mainly reached through algebraic calculations. However, the application of algebraic calculations was also the most frequent approach that led to incorrect responses to the Perimeter Percentage Problem. For example some participants gave the following calculations: *"Perimeter of rhombus =4a, Perimeter of parallelogram =2(2a+a/2)=5a"*. There were also a number of prospective teachers who did not provide any justification for their solution. In the Perimeter Multiplicative Problem several participants incorrectly claimed that the perimeter of the parallelogram is equal to that of the rhombus. Some of their justifications for providing this erroneous solution were: *"Because we divide one side by 6 and multiply the other by 6"*, *"Because we divide one side with what we have multiplied the other side"*, *"The two sides are increased with the same amount that other two sides are decreased"*. These justifications are in line with the intuitive rule *Same A – Same B*. There were also some participants who did not provide any justifications for their response or they had tried to answer the problem by solving it with specific numbers.

In the Rhombus Area Problems we have a different picture, the percentage of correct responses for the Additive, Percentage and Multiplicative Area Problems decline. Specifically, the percentages of correct responses are, for the Area Additive Problem 49%, for the Area Percentage Problem 45% and for the Area Multiplicative Problem 55%. These percentages of correct responses are lower than the percentages of correct responses for all other problems investigated in this study. The correct responses to these problems were achieved mainly through algebraic calculations. For the Area Additive Problem almost 10% of the participants gave the incorrect solution that the area of the parallelogram is equal to the area of the rhombus. Most of the participants that gave this solution provided justifications that are in line with the intuitive rule *Same A – Same B*, *"What is subtracted by one side is added to the other one. Therefore, the product of a×b will be the same"*, *"The more the two sides increase the more the other two decrease"*, *"Because we increase and decrease the sides of the parallelogram by the same numbers"*. In a similar fashion, in the Area Percentage Problem almost 15% of the prospective primary school teachers argued that the areas of two shapes are equal. Some based these solutions to intuitive justifications, for example *"we increase the one side by 50% and decrease the other by 50%, therefore the same surface is covered"*, *"It is equal because we increase the two sides and at the same time we decrease the other two. Therefore, the sides become equal"; "we multiply it and then divide it by the same number"*. Of course, there were also students that did not provide any justifications for their solutions. In the Area Multiplicative Problem 10% of the participants gave the incorrect response that the area of the parallelogram is larger than the area of the rhombus. Most of the participants, who claimed this, again did not provide any justifications for their answers.

Especially noteworthy in the findings presented in Table 2, is the fact that in the Rhombus Area Problems a large percentage of the participants either claimed that the relationship between the area of the parallelogram and the rhombus *"Cannot be determined"* or they could not provide an answer; a phenomenon that did not occur for the Square Area Problems. Specifically, almost a quarter of the participants (24,8%) claimed that they could not determine the answer to the Area Additive Problem because of missing information, whereas 14% and 18% argued that the answers could not be determined for the Area Percentage Problem and the Area Multiplicative Problem respectively. Some of the prospective teachers justified their *"Cannot determine"* solutions by providing general claims such as *"we need more evidence to find the area of the shapes"*. Others specified the elements they felt that they were missing, *"we don't know the height of the parallelogram"* or *"we don't* *know the diagonals of the rhombus"*. There were also some participants who clearly claimed that they "*could not remember the formula*". It is also important that if we compare the results in Table 1 (Square/Rectangle) to the results in Table 2 (Rhombus/Parallelogram), we detect an increase in the number of participants that did not provide an answer to the problems (3%, 6%, 4,5%, 13,5%, 21,1% and 14,3%). This increase is even larger for the last three problems i.e. the Rhombus/Parallelogram Area Problems.

RHOMBUS – PARALLELOGRAM PROBLEMS						
	Perimeter (%) N=133			Area (%) N=133		
Solution	Additive	Percentage	Multiplicative	Additive	Percentage	Multiplicative
Smaller	2,3	2,3	2,3	48,9*	45,1*	10,5
Equal	93,2*	82*	9,8	9,8	14,3	54,9*
Larger	1,5	9	82,7*	3	6	3
Not Determ.	0	0,8	0,8	24,8	13,5	17,3
No answer	3	6	4,5	13,5	21,1	14,3
** marks the percentage of the correct solution*						

Table 2: Students' responses to the Rhombus-Parallelogram Problem

Discussion

This paper investigated Cypriot elementary school prospective teachers' tendency to provide solutions in line with the intuitive rule *Same A - Same B* when solving *Additive, Percentage and Multiplicative - Comparison-Problems* regarding areas and perimeters.

The questionnaires used in this study included twelve problems. The correct answers to all these Problems could be determined by algebraic calculations. As expected, a great proportion of the participating Cypriot elementary school prospective teachers were able to respond correctly to the first part of the questionnaire which included the Square/Rectangle Problems. The justifications they presented were mainly based on algebraic calculations. There were, however, some participants who gave incorrect responses. The percentage of incorrect responses was greater for the Area Problems rather than for the Perimeter Problems. In addition to this, the Percentage Area Problem was the one with the highest percentage of incorrect responses. It was also found that a number of prospective teachers' incorrect solutions could be interpreted in light of the intuitive rules theory and more specifically, with the intuitive rule *Same A - Same B*. There was also some small evidence of responses which were mainly consistent with the intuitive *More A - More B* reasoning.

In the second part of the questionnaire the same problems were presented to the participants with the difference being that the square and rectangle were replaced by rhombus and parallelogram. By comparing the correct responses in the Square/Rectangle Perimeter Problems to the Rhombus/Parallelogram Perimeter Problems we observe a small decrease of correct responses. Almost all of the incorrect solutions to these problems were given based on incorrect calculations or *Same A – Same B* arguments. An especially interesting finding of this study was the sharp increase of incorrect solutions, as well as the rise in the number of participants that were unable to respond that appeared in the Rhombus/Parallelogram Area

Problems.

Overall, the results suggested that the Square/Rectangle Problems were the easiest for the Prospective Teachers. The Perimeter Problems were easier than the Area Problems for both the Square/Rectangle and Rhombus/Parallelogram Problems. In addition to this, the results also suggested that the Percentage Problems were the most problematic ones. Thus, it can be argued that as individuals move on to more unfamiliar problems and the number of incorrect responses increases, the evidence of the influence of the intuitive rule *Same A – Same B* becomes more apparent.

Conclusion

Summing up, knowledge of prospective elementary school teachers' common errors and of the sources of these errors is of great importance for mainly two reasons. First, prospective teachers will soon start their teaching careers and thus need to have solid content knowledge. Therefore, they need to be aware of their own possible mathematical errors. Second, teachers also need to have a good pedagogical knowledge, thus not simply be able to identify students' errors but also be in a position to understand why they have occurred and possibly find ways in which they can help students to overcome them.

This study corroborates with a number of other studies that provide evidence of the impact of the intuitive rule *Same A – Same B* in directing individuals' reasoning. Its impact appeared to increase as participants were asked to respond to more challenging problems. Still, we feel much more work is needed. We believe that it is important to continue investigating the impact of the intuitive rules on Cypriot prospective teachers as well as on students' mathematical tasks. Such evidence can be used when designing meaningful teaching activities.

Moreover, since the intuitive rules theory is only one theoretical model for interpreting and analyzing students' mathematical solutions, it is important to try to analyze the data with the use of additional theoretical models that may shed more light on students' mathematical reasoning.

References

Brousseau, G. (1997). *Theory of didactical situations in mathematics*, Kluwer Academic Publisher.

Dembo, Y., Levin, I., & Siegler, R.S. (1997). A comparison of the geometric reasoning of students attending Israeli ultra-orthodox and mainstream schools. *Developmental Psychology, 33,* 92-103.

Fischbein, E. (1987). Intuition in science and mathematics: An educational approach. Dordrecht, Netherlands: Reidel.

Hirstein, J. (1981). The second area assessment in mathematics: Area and volume, *Mathematics Teacher, 74,* 704-708.

Hoffer A.R., & Hoffer, S.A.K. (1992). *Geometry and visual thinking.*. In T.R. Post, (Ed.), Teaching mathematics in grades k-8: Research-based methods (2nd Ed), Boston: Allyn and Bacon.

Medici, D., Marchetti, P., Vighi, P., & Zaccomer, E. (submitted) Comparing perimeters and areas children's preconceptions and spontaneous procedures.

Proceedings of the Conference for the European Society for Research in Mathematics Education. Sant Feliu de Guíxols, Spain

Menon, R. (1998). Preservice teachers' understanding of perimeter and area. *School Science and Mathematics, 98*(7), 361-368.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and Standards for School Mathematics*, Reston, VA: NCTM.

Reinke, K.S. (1997). Area and perimeter: Prospective teachers' confusion. *School Science and Mathematics, 97*(2), 75-77.

Shulman, L.S. (1986). Those who understand: Knowledge growth with teaching. *Educational Researcher, 15*(2), 4-14.

Stavy, R., & Tirosh, D. (1993). Subdivision processes in mathematics and science. *Journal of Research in Science Teaching, 30,* 579-586.

Stavy, R., & Tirosh, D. (2000). *How students' (mis-)understand science and mathematics: Intuitive rules.* New York: Teachers College Press.

Tirosh, D., & Graeber, A., (2003). Challenging and changing mathematics teaching classroom practices. In A.J. Bishop, M. A. Clements, C. Keitel, J. & Leung, F. K. Leung (Eds.), *Second international handbook of mathematics education* (pp. 643-687). Dordrecht: Kluwer.

Tirosh, D., & Stavy, R. (1999). Intuitive rules: A way to explain and predict students' reasoning. *Educational Studies in Mathematics, 38,* 51-66.

Tsamir, P., & Mandel, N. (2000). The intuitive rule same A – same B: The case of area and perimeter. In T. Nakahara, & M. Koyama (Eds.), *Proceedings of the 24th Annual Meeting for the Psychology of Mathematics Education,* Hiroshima: Japan. (Vol. IV, pp. 225-232).

Tsamir, P., Tirosh, D., Stavy, R., & Ronen, I. (2000). The intuitive rules theory, A diagnostic tool in mathematics and science education. In G. Makrides and A. Gagatsis (eds.) *Proceedings of the 2nd Mediterranean Conference on Mathematics Education* (Vol I, pp. 327-346). Nicosia: Cyprus.

Vergnaud, G. (1998). A comprehensive theory of representation for mathematics education. *Journal of mathematical behavior, 17*(2), pp.167-181.

Vosniadou, S., (2002). Exploring the Relationships between Conceptual Change and Intentional Learning. In G.M. Sinatra and P.R. Pintrich (Eds). *Intentional Conceptual Change.* Mahwah, NJ: *Lawrence Erlbaum Associates*, Inc.

Walter, N. (1970). A common misconception about area. *Arithmetic Teacher, 17,* 286-289.

Woodward, E., & Byrd, F. (1983). Area: Included topic, neglected concept. *School Science and Mathematics, 83,* 343-347.

Zazkis, R. (1999). Intuitive rules in number theory: Example of "The more of A, the more of B" rule implementation. *Educational Studies in Mathematics, 40*(2), 197-209.