Design and implementation of discussion based instruction for 'ratio and proportion'

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Abstract

Selected results from a case study on the teaching of 'ratio' in a primary school classroom (pupils aged 10-11) are presented in this paper. Two teaching sessions were designed during the meetings of one 'teachers' inquiry group' and then implemented by a member of the group in his class. Both sessions were built around selected 'ratio tasks' and were characterized by two essential features: the prominence of the pupils' argumentation through discussion and the use of appropriate models. By offering an outline of one of the teaching sessions, enriched with parts of representative episodes, it is suggested that such a didactic approach could aid the pupils to reason proportionally.

Key words

Ratio, proportion, proportional reasoning, class discussion, models

Introduction

The pupils' ability to handle successfully mathematical problems related to 'ratio' and 'proportion' (i.e. the manifestation by pupils of 'proportional reasoning') is considered critical not only in studying mathematics but other subjects as well. Nevertheless, as demonstrated in Misailidou and Williams (2003), young pupils experience difficulties when dealing with such problems: instead of reasoning proportionally, very often, they use the 'additive strategy' to obtain an answer. When using such a strategy to solve a ratio item, 'the relationship within the ratios is computed by subtracting one term from another, and then the difference is applied to the second ratio' (Tourniaire & Pulos 1985, p.186).

The pupils' difficulties indicate that more 'effective' teaching approaches are needed. Previous research concerning 'small group collective argumentation' (Misailidou and Williams, 2004) proposed teaching 'tools' for enhancing pupils' proportional reasoning.

This paper reports an attempt to implement the teaching ideas, which were generated in small group environments, in a primary school classroom. More specifically, results from a teaching session focused on a central 'ratio' task are presented and discussed. This teaching session was part of a case study concerning the teaching of 'ratio' in a primary school classroom consisting of 29 pupils (aged 10-11).

Theoretical framework

Bell, Swan, Onslow and Purdy (1985) stress that the children's strategies and most importantly their errors can be used as a starting point for effective mathematics teaching. The same authors suggest that after significant errors are identified, they can be resolved through discussion and appropriate argumentation. Pesci (1998) supports these ideas in the context of proportional reasoning: she notes that the pupils' errors are opportunities for mathematical investigation and not something that should be hidden or dismissed.

Accordingly, in research preceding this study, a diagnostic test for ratio and proportion was used for exposing the pupils' strategies and errors in varying 'ratio' items (Misailidou and Williams, 2003). Selected problems from that test were used as central tasks in a subsequent study of problem solving in small groups of pupils (Misailidou and Williams, 2004 and 2004b). The role of discussion and of the generation of arguments was recognised as crucial in aiding the pupils' proportional reasoning (Pesci, 1998) in such problem solving activities. In addition, the provision of appropriate 'tools' is reported to be significant in mathematics teaching and learning in general (Baturo, Cooper and Thompson, 2003). Thus, 'tools' for facilitating the pupils' arguments in discussing the 'ratio tasks' were designed and used.

It has been reported that the mathematics teachers' participation in 'communities of practice' is an effective strategy for their professional development (Gomez, 2002). Thus, the results of the two studies mentioned above were communicated to a 'teachers' inquiry group'('TIG'): a group consisting of teachers and researchers who met and worked together with the aim of developing effective teaching practice. Such a group was seen as the link between the research in controlled small group environments and the implementation of its findings in authentic classroom environments.

Methodology

The author participated in all TIG meetings during one academic year. The findings of the author's research were communicated and discussed within the TIG. Jack, a teacher-member of the group, decided to introduce the topic of 'ratio' in his class based on these findings.

Two teaching sessions were planned. Each session was built around a central task derived from the ratio and proportion diagnostic test: a 'mixing paint' task (called Paint) and a 'sharing bread' task (called Campers). Jack was aware of the variety of strategies that his pupils would use in attempting to solve these tasks since the diagnostic test had already been administered in his class; hence, he planned his teaching accordingly.

Two kinds of models were used to aid the pupils' argumentation: pictorial models which were suggested by the author and concrete models (coloured counters on an overhead projector) which were the 'product' of the TIG discussions. The aim, set by the TIG, for both of the sessions was to let the pupils work individually, in small groups and as a whole class and each time to 'guide' them in persuading each other through reasonable arguments about methods and answers concerning the tasks.

The two sessions were videotaped and transcribed. Copies of all the children's written work were collected and field notes summarizing the classroom events were written after each session.

Both of the teaching sessions were analysed drawing on discourse analysis and sociocultural theories of learning (Gee, 1996). First, a comparative analysis identified their common structural characteristics. Then the analysis focused on each session separately. The transcribed text was segmented to signal the changes in (a) the teacher's guidance and actions and (b) the nature of pupils' work. In these segments 'episodes' were identified (and coded accordingly) reflecting changes in the nature of (a) argumentation and (b) the use of 'tools'. Selected results from this analysis are presented below.

Results

At the beginning of the teaching session Jack presented to his class the Campers problem by using an overhead projector. The problem was presented as follows:

> *10 campers have camped at the 'Blue Mountain' camp the previous week.*
>
> *Each day there are 8 loaves of bread available for them to eat.*
>
> *The loaves are provided by the camp's cook and the campers have to share the bread equally amongst them.*
>
> *This Monday 15 campers camped at the 'Blue Mountain' camp.*
>
> *How many loaves are there available for them for the day?*

Then Jack presented to his class a 'pictorial model' for the problem that was used in the author's earlier research as a tool for facilitating the pupils' proportional reasoning during group discussions (Misailidou and Williams, 2004b). The same model was replicated in another study (Elia and Philippou, 2004) and used during individual interviews: the students of that study also acted on the model and used proportional reasoning for solving the *Campers* problem. This model is presented in Figure 1.

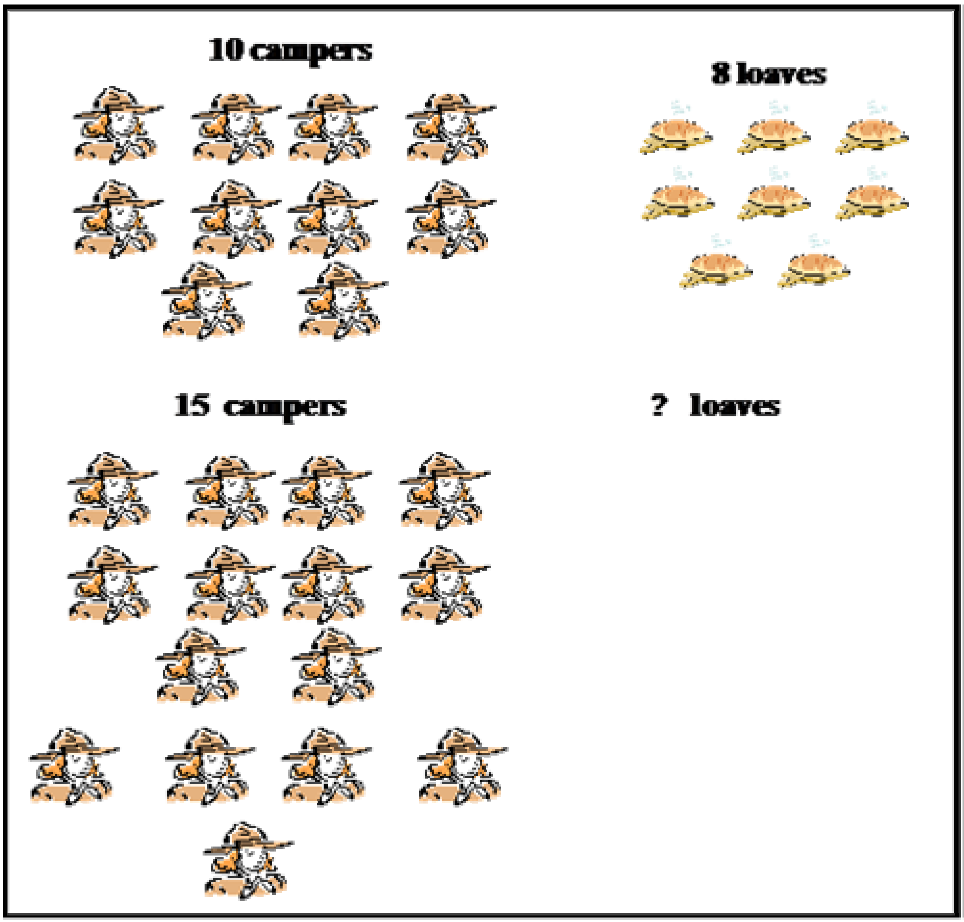

Figure 1: The pictorial model for the Campers item

The pupils were arranged in the class so as to form 6 groups. Initially, though, Jack asked the pupils to work individually and ‘draw’ a solution for the problem. On completion of the individual work, Jack ‘legitimized’ the existence of different ways of approaching the problem: he asked the pupils to present their answers and methods to their groups. Each group would have to come to a consensus on an answer and method that they then would present to the whole class. Jack encouraged each group to represent the agreed answer on a small ‘group whiteboard’ by using whatever forms of representation they wanted: ‘words, pictures, letters and numbers where possible’. One ‘group whiteboard’ is presented in Figure 2.

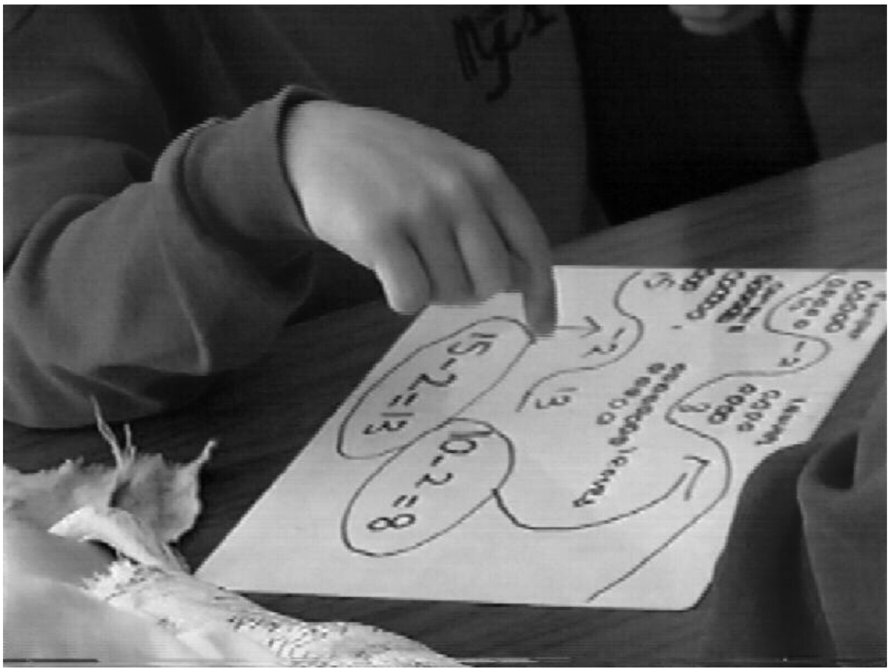

Figure 2: The pupils’ work on the whiteboard

The following different answers for the problem were agreed within the different groups:

- The answer ‘13’ as a result of the additive strategy: since 8 loaves are available for 10 campers and 10-2=8, then 15-2=13, so 13 loaves should be available for 15 campers.
- The answer ‘11’ as a result of an incorrect application of a build up method: since 10x2=20 and 20-5=15, then 8x2=16 and 16-5=11.
- The answer ‘12’ as a result of a build up method: 10÷2=5 and 10+5=15 so 8÷2=4 and 8+4=12.

After all the groups had reached a consensus, a representative from each group presented the agreed answer to the class (by referring to the ‘group whiteboard’). Two groups advocated the answer ‘12’, three the answer ‘13’ and one the answer ‘11’.

This paper focuses on selected ‘moments’ of the discussion: moments that concern the presentation and justification of the additive strategy and the answer ‘13’ first and later, following the class discussion and the use of concrete models, the ‘change of mind’ (as demonstrated by the change of pupils’ arguments) and the adoption of the answer ‘12’.

A representative from a group that agreed on the answer ‘13’ was the first to be asked by the teacher to present their method to the class:

Sheila: We found that there are 10 campers and then we took away 2, to give us 8. So we did the same with the next one which was 15 campers. We took 2 and we got 13 loaves, and we wrote that down.

Katia: [Adressing the teacher] Why did she took 2?

Teacher: Why did you take 2?

Sheila: Because to get from 10 to 8 you got to take 2.

Katia seemed content with the answer given by Sheila but a member of another group, Jo, felt that she had to demonstrate to the class that the additive strategy ‘works’ for every case. The teacher ‘supported’ her argument by mentioning a ‘small group’ conversation moment that he had heard earlier:

Jo: If you end up with different amount of….if you had 20 campers and you took 2, you have 18 loaves.

Teacher: [Adressing Anna, a member from another group] Is that what you were talking about…you were happy to talk about as many campers as you want. Didn’t you?

Anna: Yeah.

Teacher: How did you go? You said…30 campers, how many loaves would you buy?

Anna: 28.

Teacher: Thank you that group. Now sit down for me Sheila.

Another interesting ‘moment’ was when a group representative presented to the class the build up method that has as a result the answer ‘12’:

Keith: We got 10 campers and 8 loaves of bread for the 10 campers. So we halved that[points to the pictures of the campers on the ‘group whiteboard] and that [points to the pictures of the loaves on the ‘group whiteboard].

Teacher: Half what?

Keith: Half the campers and the bread.

Teacher: [Anna says something to the teacher] Anna wants to know why you did that.

Chris: Um…to make it simpler.

Teacher: Make it simpler. So go on. Carry on.

Chris: Then we added half of the 10 campers and the 8 bread to it [points to the pictures of the whitebboard]…to get 15 campers and 12 loaves of bread.

Teacher: Nina?

Nina: Half 15 doesn’t give you 5.

Chris: Half 10.

One notices in the above explanation that the pictorial model affected the pupil’s discourse: he ‘integrated’ the model in his argument by using the demonstrative pronoun ‘that’ in combination with the gesture of ‘pointing’. Such discourse was exhibited by other pupils as well, during the teaching session and supports the hypothesis that an appropriate model could facilitate young pupils in expressing their ideas.

Keith’s explanation did not convince the supporters of the additive strategy to abandon it (Sheila stressed that she ‘sticked’ with ‘13’) and the discussion went on.

During the course of the discussion, the answer ‘11’ was abandoned and some supporters of the additive strategy changed their minds convinced by their peers’ arguments. Nevertheless, there were strong supporters of both the answers ‘13’ and ‘12’ until the moment that Jack introduced the ‘concrete model’. He arranged yellow and red counters on the overhead projector and introduced the model as follows:

> *Teacher: Can you just go for yellow being campers and red being bread? Is that all right? So for 10 campers [he arranges 10 yellow counters on the projector]…how many loaves of bread do I need? What was the problem?*
>
> *Tom: 8.*
>
> *Teacher: 8 [he arranges 8 red counters on the projector at the right of the yellow counters].*

Then Jack went on by ‘splitting’ the ‘campers-counters’ into two groups of 5 which led the pupils to identify that ‘5 campers get 4 loaves of bread’. The ‘grouping action’ on the concrete model continued by Jack putting an additional 5 ‘counters-campers’ on the projector and the pupils concluding that Jack needed to add ‘4 loaves on’ in order to achieve the final answer ‘12 loaves’ for the problem. In Figure 3 various snapshots are presented from the use of the concrete model in the classroom.

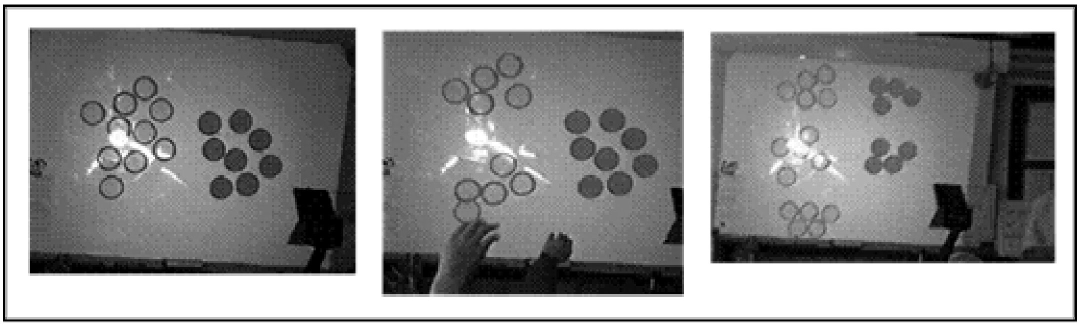

Figure 3: The use of the concrete model in the classroom

The ‘grouping action’, as demonstrated by using the concrete model, seemed to have convinced the supporters of the answer ‘13’ to reconsider their arguments: Nina admitted that for 5 campers, in order ‘to keep it fair’, 4 loaves of bread are needed. These data lead to the hypothesis that the ‘grouping action’ (afforded by the model) implicitly foregrounds the ratio ‘campers:loaves’ which is ‘5:4’ and therefore leads to a multiplicative approach to the problem.

Concluding remarks

The aim of this report was to complement the existing literature on teaching practices that support pupils’ proportional reasoning. There is already evidence in the literature that pupils who use an additive approach when solving a ‘ratio task’ could be aided in reasoning proportionally in a class environment which is characterised by the exchange of arguments in discussion. For such a discussion to be fruitful, this report isolates some necessary features:

- An appropriate problematic situation which will provoke and sustain the discussion. Such a situation can be provided by an item like *Campers*. This item provoked a variety of answers from the pupils and thus provided a rationale for discussion. Furthermore, its ‘sharing context’ afforded the ‘action of grouping’ as a predecessor of a multiplicative strategy

- ‘Tools’ that facilitate the generation of arguments in discussion by (a) making the context of the task more prominent and (b) providing a shared means of communication for the pupils. As stressed in Misailidou and Williams (2004), such tools are particularly important for young children who do not yet posses adequate mathematical terminology to communicate their thoughts and strategies. Both the pictorial representation, shown in Figure 1, and the concrete model, shown in Figure 3, meet these requirements. Additionally, the concrete model afforded the ‘action of grouping’ which seemed to have aided the pupils in changing their minds from an additive to a multiplicative approach.

Finally, it has to be stressed that the role of the teacher is crucial in ensuring that the discussion between the pupils is conducted properly and the tools are used productively. Furthermore, the position adopted by this study is that didactic research proposals can only be useful when enriched by the insights of teachers (or group of teachers) and occasionally transformed in order to meet specific teaching needs.

Acknowledgement

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